1. (a) $\frac{d}{dx}\ln x = \frac{1}{x}, \frac{d}{dx}x^2 = 2x$ (seen anywhere) A1A1

attempt to substitute into the quotient rule (do **not** accept product rule) M1

e.g. $\frac{x^2\left(\frac{1}{x}\right) - 2x\ln x}{x^4}$

correct manipulation that clearly leads to result A1

e.g. $\frac{x - 2x\ln x}{x^4}, \frac{x(1-2\ln x)}{x^4}, \frac{x}{x^4} - \frac{2x\ln x}{x^4}$

$g'(x) = \frac{1-2\ln x}{x^3}$ AG N0 4

(b) evidence of setting the derivative equal to zero (M1)

e.g. $g'(x) = 0$, $1 - 2\ln x = 0$

$\ln x = \frac{1}{2}$ A1

$x = e^{\frac{1}{2}}$ A1 N2 3

[7]

2. (a) Attempt to differentiate (M1)

$g'(x) = 3x^2 - 6x - 9$ A1A1A1

for setting derivative equal to zero M1

$3x^2 - 6x - 9 = 0$

Solving A1

e.g. $3(x - 3)(x + 1) = 0$

$x = 3 \quad x = -1$ A1A1 N3

(b) **METHOD 1**

$g'(x < -1)$ is positive, $g'(x > -1)$ is negative A1A1

$g'(x < 3)$ is negative, $g'(x > 3)$ is positive A1A1

min when $x = 3$, max when $x = -1$ A1A1 N2

METHOD 2

Evidence of using second derivative (M1)

$g''(x) = 6x - 6$ A1

$g''(3) = 12$ (or positive), $g''(-1) = -12$ (or negative) A1A1

min when $x = 3$, max when $x = -1$ A1A1 N2

[14]

3. $f'(x) = -2x + 3$

$f(x) = \frac{-2x^2}{2} + 3x + c$ (M1)

Notes: *Award (M1) for an attempt to integrate. Do not penalize the omission of c here.*

$1 = -1 + 3 + c$ (A1)

$c = -1$ (A1)

$f(x) = -x^2 + 3x - 1$ (A1) (C4)

[4]

4. (a) **METHOD 1**

$f''(x) = 3(x-3)^2$ A2 N2

METHOD 2

attempt to expand $(x-3)^3$ (M1)

e.g. $f'(x) = x^3 - 9x^2 + 27x - 27$

$f''(x) = 3x^2 - 18x + 27$ A1 N2

(b) $f'(3) = 0, f''(3) = 0$ A1 N1

(c) **METHOD 1**

f'' does not change sign at P R1

evidence for this R1 N0

METHOD 2

f' changes sign at P so P is a maximum/minimum (*i.e.* not inflexion) R1

evidence for this R1 N0

METHOD 3

finding $f(x) = \frac{1}{4}(x-3)^4 + c$ and sketching this function R1

indicating minimum at $x = 3$ R1 N0

[5]

5. (a)

	A	B	E
$f'(x)$	negative	0	negative

A1A1A1 N3

(b)

	A	B	E
$f''(x)$	positive	positive	negative

A1A1A1 N3

[6]

6. (a) $x = 4$ (A1)

g'' changes sign at $x = 4$ or concavity changes (R1) 2

(b) $x = 2$ (A1)

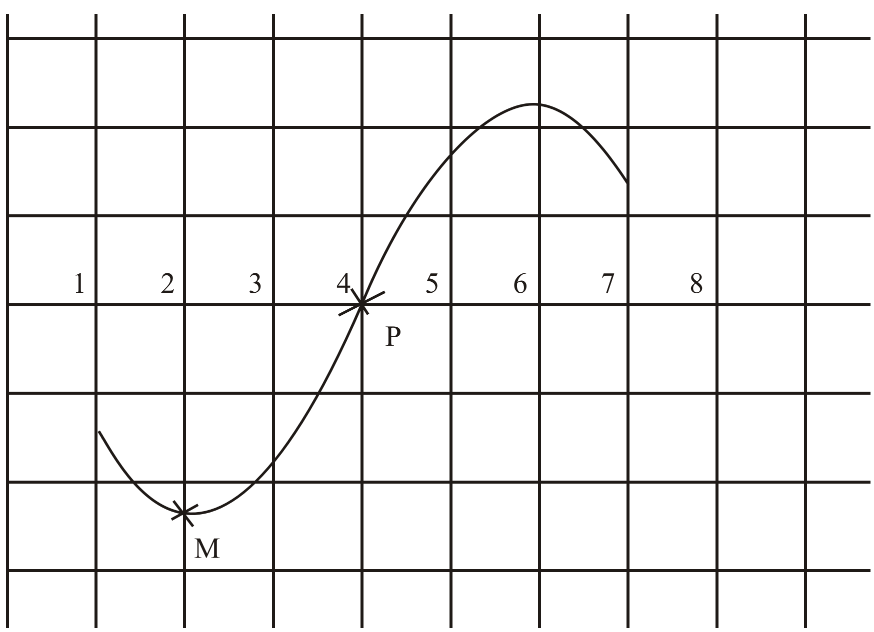
EITHER

g' goes from negative to positive (R1)

OR

$g'(2) = 0$ **and** $g''(2)$ is positive (R1) 2

(c)



(A2)(A1)(A1) 4

Note: *Award (A2) for a suitable cubic curve through (4, 0), (A1) for M at x = 2, (A1) for P at (4, 0).*

[8]

7. (a) **METHOD 1**

$f'(x) = -6 \sin 2x + 2 \sin x \cos x$ A1A1A1

$= -6 \sin 2x + \sin 2x$ A1

$= -5 \sin 2x$ AG N0

METHOD 2

$\sin^2 x = \frac{1-\cos 2x}{2}$ (A1)

$f(x) = 3 \cos 2x + \frac{1}{2} - \frac{1}{2} \cos 2x$ A1

$f(x) = \frac{5}{2} \cos 2x + \frac{1}{2}$ A1

$f'(x) = 2\left(\frac{5}{2}\right)(-\sin 2x)$ A1

$f'(x) = -5 \sin 2x$ AG N0

(b) $k = \frac{\pi}{2}$ $(=1.57)$ A2 N2

[6]

8. (a) $h = 3$ (A1)

$k = 2$ (A1) 2

(b) $f(x) = -(x-3)^2 + 2$

$= -x^2 + 6x - 9 + 2$ (must be a correct expression) (A1)

$= -x^2 + 6x - 7$ (AG) 1

(c) $f'(x) = -2x + 6$ (A2) 2

(d) (i) tangent gradient $= -2$ (A1)

gradient of $L = \frac{1}{2}$ (A1)(N2) 2

(ii) **EITHER**

equation of L is $y = \frac{1}{2}x + c$ (M1)

$c = -1$. (A1)

$y = \frac{1}{2}x - 1$

OR

$y - 1 = \frac{1}{2}(x - 4)$ (A2) (N2) 2

(iii) **EITHER**

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (M1)

$2x^2 - 11x + 12 = 0$ (may be implied) (A1)

$(2x - 3)(x - 4) = 0$ (may be implied) (A1)

$x = 1.5$ (A1) (N3) 4

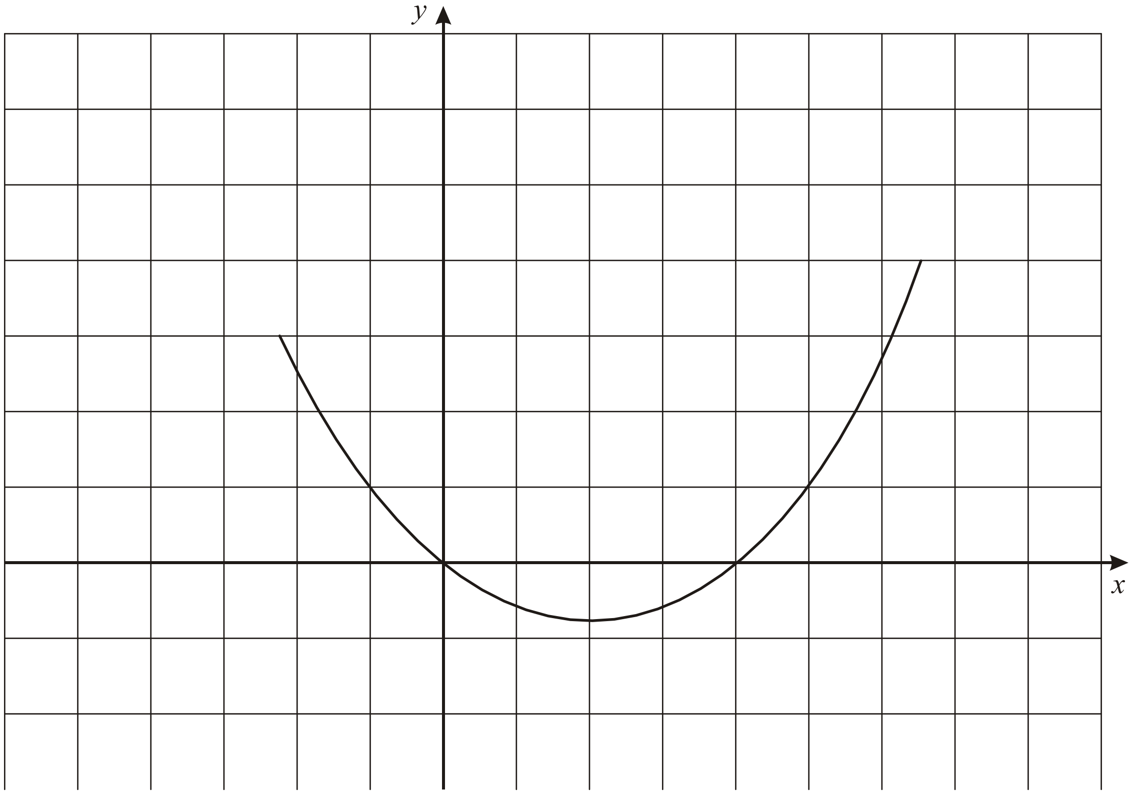
OR

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (or a sketch) (M1)

$x = 1.5$ (A3)(N3) 8

[13]

9.



(A2)(A1)(A1)(A2) (C6)

Note: *Award A2 for correct shape (approximately parabolic), A1 A1 for intercepts at 0 and 4, A2 for minimum between $x = 1.5$ and $x = 2.5$.*

[6]

10. (a) (i) $f(x) = \dfrac{2x+1}{x-3}$

$= 2 + \dfrac{7}{x-3}$ by division or otherwise (M1)

Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR $\lim\limits_{x\to\infty} \dfrac{2x+1}{x-3} = 2$ (M1)(A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR make x the subject

$yx - 3y = 2x + 1$

$x(y-2) = 1 + 3y$ (M1)

$x = \dfrac{1+3y}{y-2}$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

Note: *Accept inexact methods based on the ratio of the coefficients of x.*

(ii) Asymptote at $x = 3$ (A1)

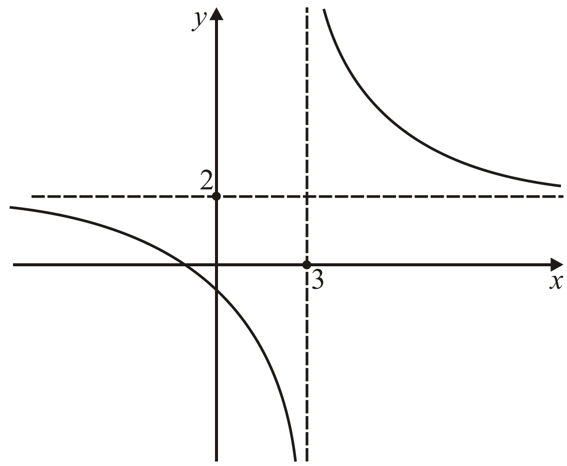
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\dfrac{1}{2}$ $\left(-\dfrac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\dfrac{1}{3}$ $\left(0, -\dfrac{1}{3}\right)$ (M1)(A1) 4

Note: *These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
"Shape" (A2).

(d) $f'(x) = \dfrac{(x-3)(2)-(2x+1)}{(x-3)^2}$ (M1)

$= \dfrac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)
And $f(4) = 9$ ie $S(4, 9)$ (A1)
$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)
$7x + y - 37 = 0$ (A1) 6

(e) at T, $\dfrac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x-3)^2 = 1$ (A1)
$x - 3 = \pm 1$ (A1)

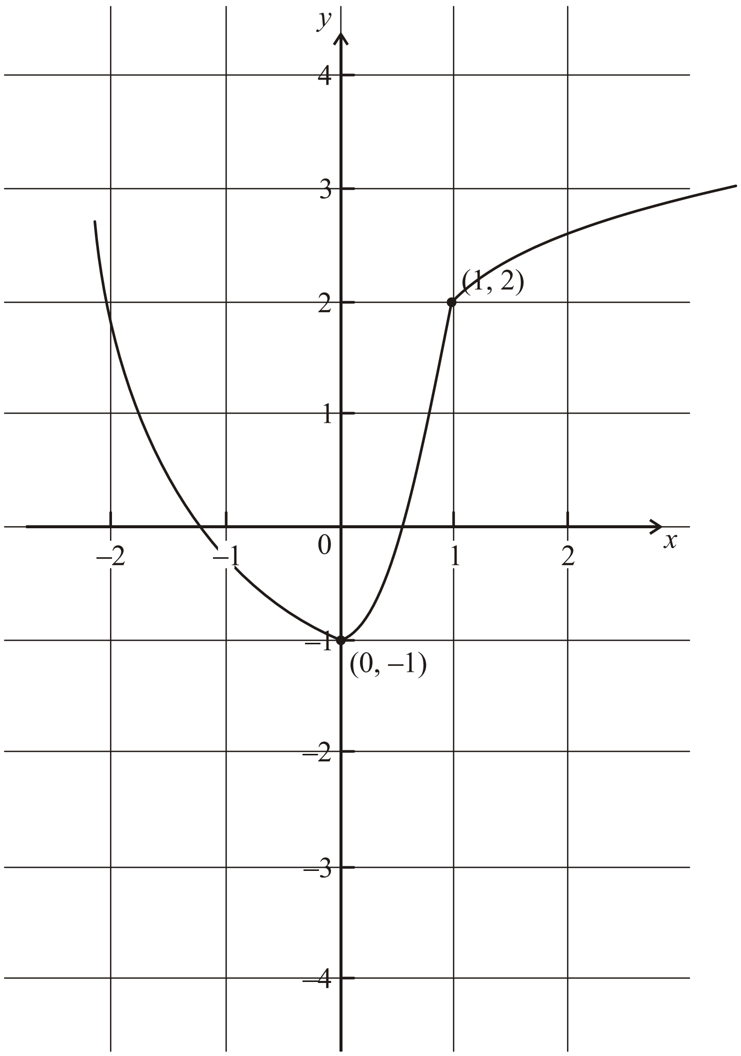
$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\} \begin{array}{l} S(4, 9) \\ T(2, -5) \end{array}$ (A1)(A1) 5

(f) Midpoint $[ST] = \left(\dfrac{4+2}{2}, \dfrac{9-5}{2}\right)$

$= (3, 2)$
= point P (A1) 1

[24]

11.



A1A1A1A1A1A1 N6

Notes: *On interval [– 2,0], award A1 for decreasing, A1 for concave up.*

On interval [0,1], award A1 for increasing, A1 for concave up.

On interval [1,2], award A1 for change of concavity, A1 for concave down.

[6]

12. $y = x^3 + 1$

$\frac{dy}{dx} = 3x^2$

= Slope of tangent at any point

Therefore at point where $x = 1$, slope = 3 (M1)

$\Rightarrow$ Slope of normal $= -\frac{1}{3}$ (M1)(A1)

$\Rightarrow$ Equation of normal: $y - 2 = -\frac{1}{3}(x - 1)$

$3y - 6 = -x + 1$

$x + 3y - 7 = 0$ (A1) (C4)

Note: *Accept equivalent forms eg* $y = -\frac{1}{3}x + 2\frac{1}{3}$

[4]

13. evidence of choosing the product rule (M1)

$f'(x) = e^x \times (-\sin x) + \cos x \times e^x$ $(= e^x \cos x - e^x \sin x)$ A1A1

substituting π (M1)

e.g. $f'(\pi) = e^\pi \cos \pi - e^\pi \sin \pi$, $e^\pi(-1 - 0)$, $-e^\pi$

taking negative reciprocal (M1)

e.g. $-\frac{1}{f'(\pi)}$

gradient is $\frac{1}{e^\pi}$ A1 N3

[6]

14. (a) (i) -1.15, 1.15 A1A1 N2

(ii) recognizing that it occurs at P and Q (M1)

e.g. $x = -1.15$, $x = 1.15$

$k = -1.13$, $k = 1.13$ A1A1 N3

(b) evidence of choosing the product rule (M1)
e.g. $uv' + vu'$

derivative of x^3 is $3x^2$ (A1)

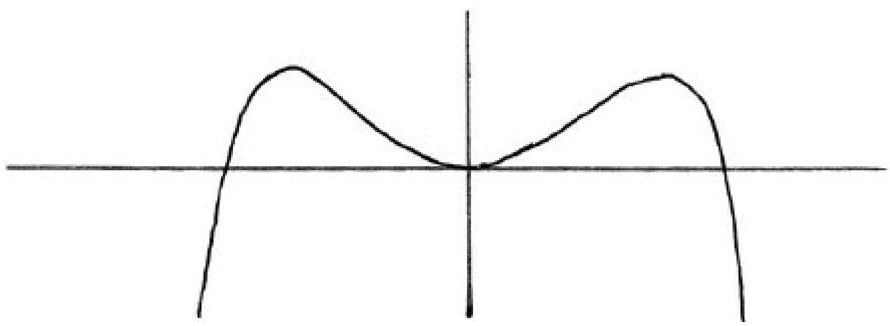
derivative of $\ln(4 - x^2)$ is $\frac{-2x}{4-x^2}$ (A1)

correct substitution A1

e.g. $x^3 \times \frac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2$

$g'(x) = \frac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2)$ AG N0

(c)



A1A1 N2

(d) $w = 2.69$, $w < 0$ A1A2 N2

[14]

15. (a) valid approach R1
e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f

-0.114, 0.364 (accept $(-0.114, 0.811)$ and $(0.364, 2.13)$) A1A1 N1N1

(b) **METHOD 1**

graph of g is a quadratic function R1 N1
a quadratic function does not have any points of inflexion R1 N1

METHOD 2

graph of g is concave down over entire domain R1 N1
therefore no change in concavity R1 N1

METHOD 3

$g''(x) = -144$ R1 N1
therefore no points of inflexion as $g''(x) \neq 0$ R1 N1

[5]

16. substituting $x = 1$, $y = 3$ into $f(x)$ (M1)
$3 = p + q$ A1

finding derivative (M1)
$f'(x) = 2px + q$ A1
correct substitution, $2p + q = 8$ A1

$p = 5$, $q = -2$ A1A1 N2N2

[7]

17. (a) gradient is 0.6 A2 N2

(b) at R, $y = 0$ (seen anywhere) A1
at $x = 2$, $y = \ln 5$ $(= 1.609...)$ (A1)
gradient of normal $= -1.6666...$ (A1)
evidence of finding correct equation of normal A1

e.g. $y - \ln 5 = -\frac{5}{3}(x - 2)$, $y = -1.67x + c$

$x = 2.97$ (accept 2.96) A1

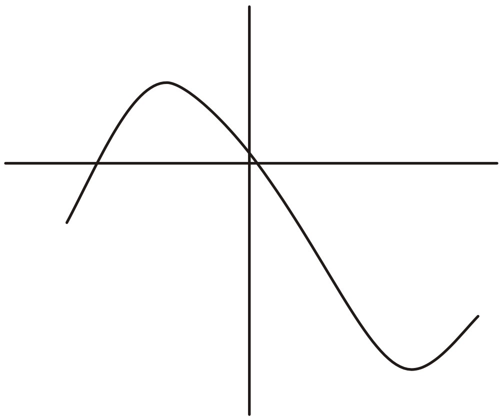
coordinates of R are (2.97, 0) N3

[7]

18. (a) **EITHER**

Recognizing that tangents parallel to the x-axis mean maximum and minimum (may be seen on sketch) R1

Sketch of graph of f M1



OR

Evidence of using $f'(x) = 0$ M1

Finding $f'(x) = 3x^2 - 6x - 24$ A1

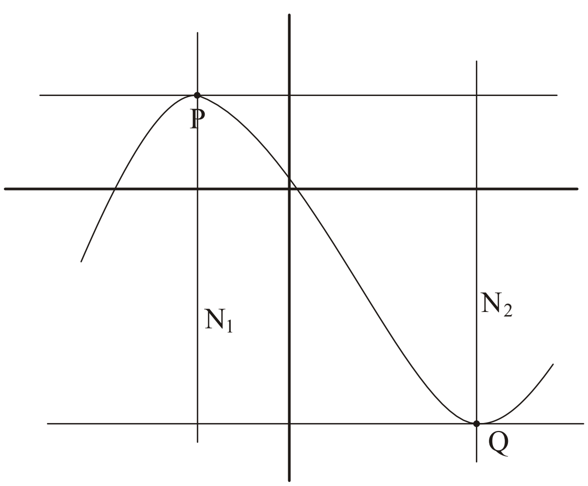
$3x^2 - 6x - 24 = 0$

Solutions $x = -2$ or $x = 4$

THEN

Coordinates are P(–2, 29) and Q(4, –79) A1A1 N1N1

(b)



(i) (4, 29) A1 N1

(ii) (–2, –79) A1 N1

[6]

19. (a) (i) $x = -\frac{5}{2}$ (A1)

(ii) $y = \frac{3}{2}$ (A1) 2

(b) By quotient rule (M1)

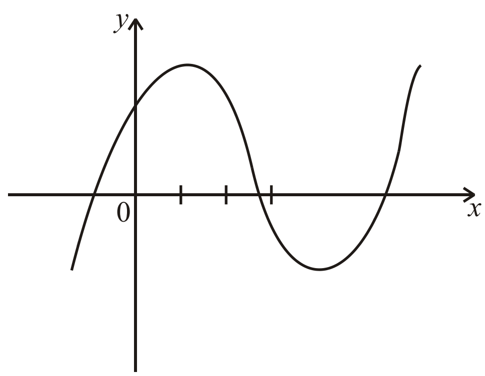
$\frac{dy}{dx} = \frac{(2x+5)(3)-(3x-2)(2)}{(2x+5)^2}$ (A1)

$= \frac{19}{(2x+5)^2}$ (A1) 3

(c) There are no points of inflexion. (A1) 1

[6]

20. **METHOD 1**



Using gdc coordinates of maximum are
(0.667, 26.9) (G3)(G3) (C6)

METHOD 2

At maximum $\frac{dy}{dx} = 3x^2 - 20x + 12 = 0 = (3x-2)(x-6)$ (M1)(A1)(M1)

$\Rightarrow$ $x = \frac{2}{3}$ must be where maximum occurs (A1)

$x = \frac{2}{3} \Rightarrow y = \left(\frac{2}{3}\right)^3 - 10\left(\frac{2}{3}\right)^2 + 12\left(\frac{2}{3}\right) + 23 = \frac{725}{27}$ (= 26.9, 3 sf) (M1)(A1)

Maximum at $\left(\frac{2}{3}, \frac{725}{27}\right)$ (C4)(C2)

[6]

21. $y = \sin(2x - 1)$

$\frac{dy}{dx} = 2\cos(2x-1)$ (A1)(A1)

At $\left(\frac{1}{2}, 0\right)$, the gradient of the tangent $= 2\cos 0$ (A1)

$= 2$ (A1) (C4)

[4]

22. (a) $f(1) = 3$ $f(5) = 3$ (A1)(A1) 2

(b) **EITHER** distance between successive maxima = period (M1)

$= 5 - 1$ (A1)

$= 4$ (AG)

OR Period of $\sin kx = \frac{2\pi}{k}$; (M1)

so period $= \frac{2\pi}{\frac{\pi}{2}}$ (A1)

$= 4$ (AG) 2

(c) **EITHER** $A\sin\left(\frac{\pi}{2}\right) + B = 3$ and $A\sin\left(\frac{3\pi}{2}\right) + B = -1$ (M1) (M1)

$\Leftrightarrow A + B = 3, -A + B = -1$ (A1)(A1)

$\Leftrightarrow A = 2, B = 1$ (AG)(A1)

OR Amplitude $= A$ (M1)

$A = \frac{3-(-1)}{2} = \frac{4}{2}$ (M1)

$A = 2$ (AG)

Midpoint value $= B$ (M1)

$B = \frac{3+(-1)}{2} = \frac{2}{2}$ (M1)

$B = 1$ (A1) 5

Note: *As the values of A = 2 and B = 1 are likely to be quite obvious to a bright student, do not insist on too detailed a proof.*

(d) $f(x) = 2\sin\left(\frac{\pi}{2}x\right) + 1$

$f'(x) = \left(\frac{\pi}{2}\right)2\cos\left(\frac{\pi}{2}x\right) + 0$ (M1)(A2)

***Note:** Award (M1) for the chain rule, (A1) for $\left(\frac{\pi}{2}\right)$, (A1) for $2\cos\left(\frac{\pi}{2}x\right)$.*

$= \pi\cos\left(\frac{\pi}{2}x\right)$ (A1) 4

***Notes:** Since the result is given, make sure that reasoning is valid. In particular, the final (A1) is for simplifying the result of the chain rule calculation. If the preceding steps are not valid, this final mark should not be given. Beware of "fudged" results.*

(e) (i) $y = k - \pi x$ is a tangent $\Rightarrow -\pi = \pi\cos\left(\frac{\pi}{2}x\right)$ (M1)

$\Rightarrow -1 = \cos\left(\frac{\pi}{2}x\right)$ (A1)

$\Rightarrow \frac{\pi}{2}x = \pi$ or 3π or ...

$\Rightarrow x = 2$ or 6 ... (A1)

Since $0 \le x \le 5$, we take $x = 2$, so the point is $(2, 1)$ (A1)

(ii) Tangent line is: $y = -\pi(x - 2) + 1$ (M1)

$y = (2\pi + 1) - \pi x$

$k = 2\pi + 1$ (A1) 6

(f) $f(x) = 2 \Rightarrow 2\sin\left(\frac{\pi}{2}x\right) + 1 = 2$ (A1)

$\Rightarrow \sin\left(\frac{\pi}{2}x\right) = \frac{1}{2}$ (A1)

$\Rightarrow \frac{\pi}{2}x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ or $\frac{13\pi}{6}$

$x = \frac{1}{3}$ or $\frac{5}{3}$ or $\frac{13}{3}$ (A1)(A1)(A1) 5

[24]

23. $y = x^2 - x$

$\frac{dy}{dx} = 2x - 1$ = gradient at any point. (M1)

Line parallel to $y = 5x$

$\Rightarrow 2x - 1 = 5$ (M1)

$x = 3$ (A1)

$y = 6$ (A1)

Point (3, 6) (C2)(C2)

[4]