Name Coppola Date_____

Pre Calculus Practice Midterm

Part I: You may use your Calculator. **ON YOUR REAL MIDTERM there are only 25 questions, and you will answer all of them! They will be multiple Choice.** No partial credit will be given, so check your work either graphically or by redoing the problem. Place all final answers on your answer sheet.

1. Solve for x: $|2x-5|<3$

$2x-5<3$ $\quad$ $2x-5>-3$ $\quad$ $x<4$, $x>1$ $\quad$ $x\in(1,4)$

2. Find the number of possible positive roots for the function: $f(x)=2x^3-8x^2-x-4$

DROS

$- \; - \; + -$

#+: 1

#-: 2 or 0

3. Given $f(x)=3x^3-3x^2+8$, use synthetic division to find f(3)

$f(3)=62$

4. Solve for x: $|x^2+6x|=3x+18$

$x^2+6x=3x+18$ $\quad$ $x^2+6x=-3x-18$

$x^2+3x-18=0$ $\quad\therefore x=-6,3$ $\quad$ $x^2+9x+18=0$

$x=-6,\pm 3$ (check!)

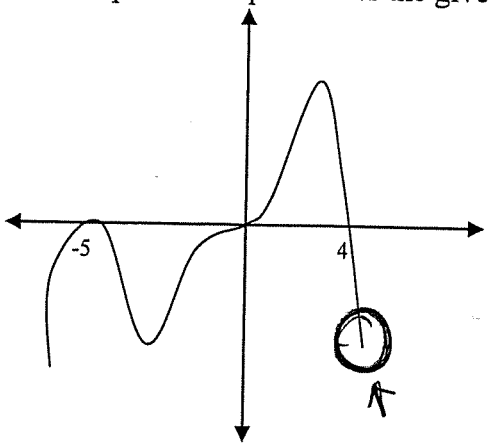
5. Find the remainder after division: $(n^3-5n^2-33n-37)\div(n-9)$

$\boxed{-10}$

6. Create a fourth degree polynomial that has roots of x=-1 and x=2

$f(x)=(x+1)^2(x-2)^2$ $\quad$ or $(x+1)(x-2)^3$ etc.

7. Write a possible equation for the given sketch.



$y=-x^3(x+5)^2(x-4)$

8. Solve for all real values of x: $x^3-3x^2-x+3=0$

$x^2(x-3)-1(x-3)=0$

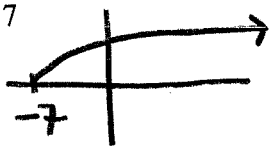
$(x^2-1)(x-3)=0$

$x=\pm 1, 3$

9. Given: $h(x) = 3x^2 - x$, find $h\left(\frac{1}{x}\right)$

$3\left(\frac{1}{x}\right)^2 - \frac{1}{x} = \frac{3}{x^2} - \frac{1\cdot x}{x\cdot x} = \boxed{\frac{3-x}{x^2}}$

10. Find the domain and range of $h(x) = \sqrt{x+7}$



Domain: $x \in [-7, \infty)$

Range: $y \in [0, \infty)$

11. Given $f(x) = x^2 + 2x$ find $\frac{f(x+h) - f(x)}{h}$

$$\frac{(x+h)^2+2(x+h)-(x^2+2x)}{h} = \frac{x^2+2xh+h^2+2x+2h-x^2-2x}{h} = \boxed{2x+h+2}$$

~~12~~. Is $f(x) = \frac{1}{3}x^6 - 2x^2 + 6$ even, odd or neither?

SKIP

13. True or false: $\sqrt{x^2-y^2} = x - y$

False!

13. Find the inverse of $\boxed{f(x) = x^3 + 1} \Rightarrow x = y^3 + 1 \Rightarrow x - 1 = y^3$

$f^{-1}(x) = \boxed{\sqrt[3]{x-1}} \Rightarrow$ graph them $\Rightarrow \boxed{y = x}$

14. Simplify: $\left(\frac{x^5 y^{-3} z}{x^{-3} y^{-4} z^8}\right)^{-2}$

$\left(\frac{x^5 x^3 y^4}{y^3 z^7}\right)^{-2} = \left(\frac{x^8 y}{z^7}\right)^{-2} = \boxed{\frac{z^{14}}{x^{16} y^2}}$

15. Simplify: $\sqrt[3]{16x^5y^8z}$ $\sqrt[3]{8 \cdot 2 \cdot x^3 \cdot x^2 \cdot y^6 \cdot y^2 \cdot z}$

$2xy^2\sqrt[3]{2x^2y^2z}$

16. List all POSSIBLE RATIONAL ROOTS of f(x): $f(x) = 3x^2 + 10x^2 + 4x - 8$

$\frac{\text{Factors of constant}}{\text{Factors of LC}} = \frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3}$

17. Factor: x^3-1

$(x-1)(x^2+x+1)$

18. Find the x and y intercepts of $f(x) = \frac{9x^2-4}{x}$

x int: $\pm 2/3$

y int: sub 0 in for x $\Rightarrow \frac{-4}{0} \Rightarrow$ NO y int

20. Factor the expression: $4v^3 - 12v^2 - 5v + 15$

$4v^2(v-3) - 5(v-3) = (4v^2-5)(v-3)$

21. Find all values for x where f(x) is undefined. $\frac{3x}{\sqrt{x-2}}$

Defined $x-2>0$, $x>2$ ∴ undef: $x \leq 2$

22. Rationalize the numerator: $\frac{\sqrt{x}-4}{2} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4}$

$\frac{x-16}{2(\sqrt{x}+4)}$

23. Factor (leave NO NEGATIVE EXPONENTS☹):

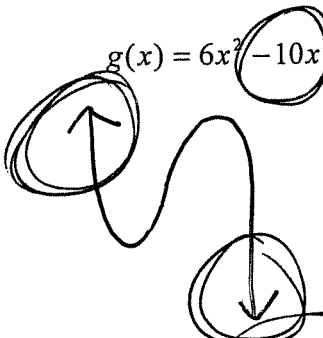
$x(1-2x)^{-3} + (1-2x)^{-2}$ $-2 + +3$

$(1-2x)^{-3}[x + (1-2x)]$

$(1-2x)^{-3}[1-x) = \frac{(1-x)}{(1-2x)^3}$

23. Describe the end behavior model of the given function.

$g(x) = 6x^3 - 10x^2 + 4x$



As $x \to -\infty$, $y \to \infty$

as $x \to \infty$, $y \to -\infty$

24. Express $x \in (-7, \infty)$ as an inequality.

$x > -7$

26. Write an equation for the graph of $y = \sqrt{x}$ after the following transformations

- Reflect over the x axis
- Shift left five units
- Shift down 3 units

$y = -\sqrt{x+5} - 3$

27. Given $f(x) = \sqrt{2x+3}$, what is the range of $f^{-1}(x)$

$x = \sqrt{2y+3}$

$x^2 \geq 2y+3$

$\frac{x^2 - 3}{2} = y$

$2x+3 \geq 0$

$x \geq -3/2$

domain of orig

R: $y \in [-3/2, \infty)$

28. True or false? $\sqrt[3]{(x-8)^4} = (x-8)^{4/3}$

29. Solve for all real values of x: $\sqrt{x} - \sqrt{x-5} = 1 \Rightarrow (-\sqrt{x-5})^2 = (1-\sqrt{x})^2$

$(\sqrt{x})^2 = (1+\sqrt{x-5})^2$

$x = 1 + 2\sqrt{x-5} + x - 5$

$4 = 2\sqrt{x-5}$

$(2)^2 = (\sqrt{x-5})^2$

$4 = x - 5$

$x = 9$

30. Use a graphing calculator to graph $f(x) = (x-1)^2(x+2)$. Approximate the INTERVALS over which f(x) is INCREASING.

Inc/ ( )

Key

Practice Midterm Part III Questions

1. $\dfrac{x(x+2)^{-5}(x+5)^{1/3}\left[x+2-6x(x+5)\right]}{x} = (x+2)^{-5}(x+5)^{1/3}\left[x+2-6x^2-\right.$

$= (x+2)^{-5}(x+5)^{1/3}\left[-6x^2+x-28\right] = \dfrac{(x+5)^{1/3}(-6x^2+8-28)}{(x+2)^5}$

2a) $f(5) = \dfrac{4}{5+3} = \dfrac{4}{8} = \dfrac{1}{2}$ b) $g(0) = 4$

c) $f(x+h) = \dfrac{4}{x+h+3}$ $f(x+h) - f(x) = \dfrac{4}{x+h+3} - \dfrac{4}{x+3}$

$= \dfrac{4(x+3) - 4(x+h+3)}{(x+h+3)(x+3)}$

$= \dfrac{4x+12-4x-4h-12}{(x+h+3)(x+3)} = \dfrac{-4h}{(x+h+3)(x+3)}$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{-4h}{(x+h+3)(x+3)}}{h}$

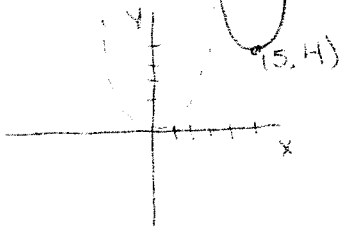
$= \dfrac{-4h}{(x+h+3)(x+3)} \cdot \dfrac{1}{h}$

$= \dfrac{-4}{(x+h+3)(x+3)}$

3. a) Reflect over x axis, shift right two, up three

b) $f(x) = \sqrt{x+4} + 1$

c)



4 $y = -(x+2)(x-1)^2(x-4)^3$

check y int: $-(2)(-1)^2(-4)^3$

$= -2 \cdot 1 \cdot -64 = +128$

~~4~~ $\frac{x+6}{x+1} - 2 < 0$

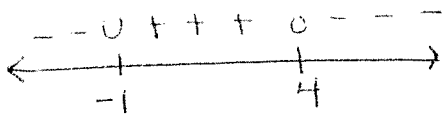
$\frac{x+6-2(x+1)}{(x+1)} < 0$

$\frac{x+6-2x-2}{(x+1)} < 0$

$\frac{-x+4}{(x+1)} < 0$

extra Q

critical points: $x = 4$ (zero), $x = -1$ (undef)



$(-\infty, -1) \cup (4, \infty)$

~~6~~ $(x-1)^{2/3} = 16$

$x - 1 = 16^{3/2}$

$x - 1 = \pm 64$

$x = 1 \pm 64 = -63, 65$

* remember: if the reciprocal has a 2 in denom, do ±

extra Q

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5) make two equations:

$x^2 - 3x = -4x + 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

$x^2 - 3x = 4x - 6$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$x = 6, 1$

$\{x = -3, 1\}$ (be sure to check! there were 2 extraneous roots!!!)

6) $2(\sqrt{x+1}) = 1 + \sqrt{2x+3}$

$(2\sqrt{x+1})^2 = (1 + \sqrt{2x+3})^2$ ← this is a binomial! Foil this!

$4(x+1) = 1 + 2\sqrt{2x+3} + 2x + 3$

$4x + 4 = 4 + 2x + 2\sqrt{2x+3}$

$-2x - 4 \quad -4 - 2x$

$2x = 2\sqrt{2x+3}$

$x = \sqrt{2x+3}$

$x^2 = 2x + 3 \rightarrow x^2 - 2x - 3 = 0 \rightarrow (x-3)(x+1) = 0$

$\{x = 3\}$ -X ✓ check!

$3 \cdot 2\sqrt{4} = 1 + \sqrt{9}$

$2 \cdot 2 = 1 + 3$ ✓

$-1 \cdot 2(0) = 1 + \sqrt{1}$

$0 \neq 2$

9. u substitution: $9t^{2/3} + 24t^{1/3} + 16 = 0$

$2(\frac{1}{3}) = 2/3$; constant

let $t^{1/3} = u$

$9u^2 + 24u + 16 = 0$

$(3u+4)(3u+4) = 0$

$u = \frac{-4}{3}$ (NOT DONE!!)

$(t^{1/3})^3 = \left(\frac{-4}{3}\right)^3$

$t = -\frac{64}{27}$

10. ~~NO~~ With or without Graphing calculator...

PRRS: $\frac{\text{Factors of constant}}{\text{Factors of LC}} = \frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

Graph it to see what will work w/ synthetic division...

$-2 \mid 3 \;\; 10 \;\; 4 \;\; -8$

$\downarrow \;\; -6 \;\; -8 \;\; 8$

$3 \;\; 4 \;\; -4 \;\; 0$

$(x+2)(3x^2+4x-4)$

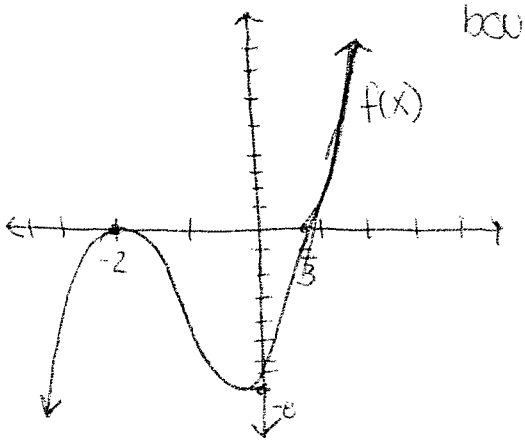
$(x+2)(3x-2)(x+2)$

$f(x) = (x+2)^2(3x-2)$

Sketch: LC: + ; Deg: odd ; EB: ↓ ↑

y int: $(2)^2(-2)$; $4 - 2 = -8$

Zeros: -2 (↑ bounce), $2/3$ (↑ st)



part III #2

a) $\frac{\text{Factors of constant}}{\text{Factors of LC}}$ = PRR's

$\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2, \pm 4} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}$

b) -4 and 2 (found on graphing calc)

c)

-4	4	4	-39	34	-8
	↓	-16	48	-36	8
	4	-12	9	-2	0

$(x+4)(4x^3-12x^2+9x-2)$

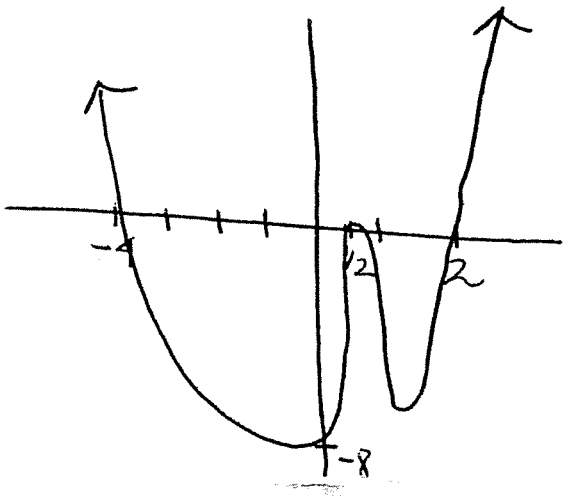
2	4	-12	9	-2
	↓	8	-8	2
	4	-4	1	0

$(x+4)(x-2)(4x^2-4x+1)$

$f(x) = (x+4)(x-2)(2x-1)(2x-1)$

$f(x) = (x+4)(x-2)(2x-1)^2$

d)



Deg: Even
LC: +
EB: ↑ ↗
zeros: $-4, 2, \frac{1}{2}$
y int: -8

#1. a) $g(f(6)) = g(6) = 1$

b) minimum of $g(x) \Rightarrow y=1, x=6$ or at the point $(6,1)$

c) x values: $(6,9)$

d) Range $g(x)$:

R: $y \in [1,7]$

e) Range $f(x)+3$

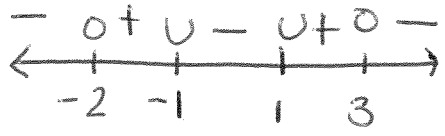
R: $y \in [-6,9]$

3. Solve: $\frac{3}{x-1}-\frac{2}{x+1}<1$ sign chart!! Kyle was here :)

$$\frac{3(x+1)-2(x-1)-1(x^2-1)}{(x+1)(x-1)}<0$$

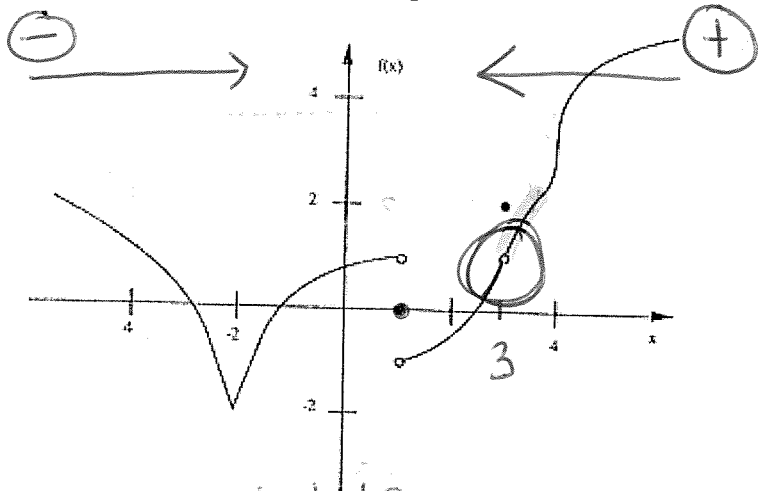
$$\frac{3x+3-2x+2-x^2+1}{(x+1)(x-1)}<0$$

$$\frac{-x^2+x+6}{(x+1)(x-1)}<0 \Rightarrow \frac{-(x-3)(x+2)}{(x+1)(x-1)}<0$$



$x\in(-\infty,-2)\cup(-1,1)\cup(3,\infty)$

4. Given the graph of the following function.



closed dots

A) $f(1)=0$ B) $\lim_{x\to 1^-} f(x)=1$ C) $\lim_{x\to 1^+} f(x)=-1$ D) $\lim_{x\to 1} f(x)$ DNE

E) $f(3)=2$ F) $\lim_{x\to 3^-} f(x)=1$ G) $\lim_{x\to 3^+} f(x)=1$ H) $\lim_{x\to 3} f(x)$ 1

I) $\lim_{x\to -2} f(x)=-2$ J) $\lim_{x\to 0} f(x)=1$ K) $\lim_{x\to -3} f(x)=0$